Name: _____ **Class:** _____ **Date:** _____ **ID: A**

Integration Review

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Evaluate the integral.

$$\int_0^5 \left(x^2 + 5\right)e^{-x}dx$$

a. $29e^{-4} + 7$
b. $-29e^{4} + 7$
c. $-42e^{-5} + 7$
d. $-29e^{-4} - 1$
e. None of these

____ 2. Evaluate the integral.

$$\int e^t \sqrt{25 - e^{2t}}\, dt$$

a. $\frac{25}{2}\arcsin\left(\frac{e^t}{5}\right) + \frac{1}{2}e^t\sqrt{25 - e^{2t}} + C$

b. $\arcsin\left(\frac{e^t}{5}\right) - \frac{1}{2}\sqrt{25 - e^{2t}} + C$

c. $\frac{25}{2}\arcsin\left(\frac{e^{2t}}{5}\right) + \frac{1}{2}\sqrt{5 - e^t} + C$

d. $25\arcsin\left(\frac{e^t}{5}\right) + \frac{1}{2}\sqrt{25 - e^{2t}} + C$

e. $\arcsin\left(\frac{e^t}{5}\right) + 25e^t\sqrt{25 - e^{2t}} + C$

____ 3. Find the integral.

$\int x^4 \ln x \ dx$

a. $\frac{1}{5}x^5(5\ln x - 1) + C$

b. $\frac{1}{5}x^4 + \frac{1}{x} + C$

c. $\frac{1}{25}x^5(5\ln x - 1) + C$

d. $\frac{1}{25}x^5(\ln x - 1) + C$

____ 4. Evaluate the integral.

$\int 7x \ln(1+x)dx$

a. $\frac{7}{4}\left(2(x^2-1)\ln(1+x) + x(x-2)\right) + C$

b. $\frac{7}{4}\left(2(x^2-1)\ln(1+x) - x(x-2)\right) + C$

c. $7\left(\frac{1}{4}x^2 - \frac{1}{2}\ln(1+x) + \frac{x^2}{2}\left(\ln(1+x)\right)\right) + C$

d. $7\left(-\frac{1}{3}x + \frac{1}{6}x^2 - \frac{1}{9}x^4 + \frac{1}{3}\ln(1+x)\right) + C$

e. $7\left(\frac{1}{6}x^2 - \frac{1}{9}x^3 + \frac{x^3}{3}\ln(1+x) + \frac{1}{3}\ln(1+x)\right) + C$

____ 5. Evaluate the integral using integration by parts with the indicated choices of u and dv.

$\int 7\theta\cos\theta d\theta,\ u = 7\theta\ , dv = \cos\theta d\theta$

a. $7\theta\cos\theta + 7\sin\theta + C$
b. $7\sin\theta + 7\cos\theta + C$
c. $7\sin\theta - 7\cos\theta + C$
d. $7\theta\sin\theta + 7\cos\theta + C$
e. None of these

____ 6. Find the integral.

$\int \cos^3 x \sin^2 x\ dx$

a. $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

b. $-\frac{1}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

c. $\sin x - \frac{1}{3}\sin^3 x + C$

d. $-\sin x + \frac{1}{3}\sin^3 x + C$

____ 7. Find the integral.

$\int \sin^3 x \cos^4 x\ dx$

a. $\frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C$

b. $-\frac{1}{7}\cos^7 x + \frac{1}{5}\cos^5 x + C$

c. $-\frac{1}{5}\cos^5 x + \frac{1}{3}\cos^3 x + C$

d. $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

____ 8. Evaluate the integral.

$\int x \sin 2x \cos 2x dx$

a. $-\frac{1}{4}\cos x \sin x + \frac{1}{4}x + C$

b. $\frac{1}{32}[\sin 32x - 32\cos 32x] + C$

c. $\frac{1}{32}[\sin 4x - 4x\cos 4x] + C$

d. $-\frac{1}{4}x\sin^2 x + \frac{1}{4}\cos x + C$

e. none of these

____ 9. Evaluate the integral.

$\int \frac{5\sin 2x}{1+\cos^4 x} dx$

a. $-5\cos^2 2x + C$

b. $1 - 5\cos^2 x + C$

c. $5\arctan\left(\sec^2 x\right) + C$

d. $-5\arcsin(\tan x) + C$

e. None of these

____ 10. A particle moves on a straight line with velocity function $v(t) = \sin \omega t \cos^4 \omega t$. Find its position function $s = f(t)$ if $f(0) = 0$.

a. $\dfrac{1 - \cos^5 \omega t}{5\omega}$

b. $\dfrac{\cos^6 \omega t + 1}{6\omega}$

c. $\dfrac{1 - \cos^7 \omega t + 1}{7\omega}$

d. $\dfrac{\sin^4 \omega t - 1}{4\omega}$

e. $\dfrac{\sin^8 \omega t + 1}{8\omega}$

____ 11. Find the integral using an appropriate trigonometric substitution.

$$\int \frac{1}{x^2\sqrt{x^2 + 25}}\, dx$$

a. $-\dfrac{\sqrt{x^2 + 25}}{5x} + C$

b. $\dfrac{\sqrt{x^2 + 25}}{5x} + C$

c. $\dfrac{\sqrt{x^2 + 25}}{25x} + C$

d. $-\dfrac{\sqrt{x^2 + 25}}{25x} + C$

____ 12. Find the integral using an appropriate trigonometric substitution.

$$\int \frac{x^3}{\sqrt{x^2+16}}\,dx$$

a. $\frac{1}{3}(x^2+32)\sqrt{x^2+16}+C$

b. $\frac{1}{3}(x^2-32)\sqrt{x^2+16}+C$

c. $\frac{1}{3}(x^2+16)^{3/2}\sqrt{x^2+16}+C$

d. $\frac{1}{3}(x^2-16)^{3/2}\sqrt{x^2+16}+C$

____ 13. Evaluate the integral.

$$\int \frac{3dx}{\left(x^2+2x+2\right)^2}$$

a. $\frac{3}{2}\left(\tan^{-1}(x+1)+\frac{x+1}{x^2+2x+2}\right)+C$

b. $\frac{1}{2}\left(\tan^{-1}(x+2)+\frac{3}{x^2+2}\right)+C$

c. $\frac{1}{2}\left(\tan(x+1)+\frac{13}{x^2+2x+2}\right)+C$

d. $\frac{3}{2}\left(\tan(x+2)+\frac{x+1}{x^2+2x+2}\right)+C$

e. $\frac{1}{2}\left(\tan^{-1}(x+3)+\frac{1}{x^2+2x+2}\right)+C$

____ 14. Write the form of the partial fraction decomposition of the rational expression. Do not find the numerical values of the constants.

$$\frac{x^2 - x - 8}{2x^3 - 3x^2 + 6x - 9}$$

a. $\dfrac{A}{2x-1} + \dfrac{Bx+C}{x^2+3}$

b. $\dfrac{A}{2x-3} + \dfrac{Bx+C}{x^2+3}$

c. $\dfrac{A}{2x+1} + \dfrac{Bx+C}{x^2-3}$

d. $\dfrac{A}{2x+3} + \dfrac{Bx+C}{x^2-3}$

____ 15. Find the integral.

$$\int \frac{x^3 - 3x^2 + 6x - 2}{x^3 - 2x^2 + x}\,dx$$

a. $\ln\left|\dfrac{x-1}{x^2}\right| - \dfrac{1}{x-1} + x + C$

b. $\ln\left|\dfrac{x^2}{x-1}\right| - \dfrac{2}{x-1} + x + C$

c. $\ln\left|\dfrac{x^2}{x-1}\right| - \dfrac{1}{x-1} + x + C$

d. $\ln\left|\dfrac{x-1}{x^2}\right| - \dfrac{2}{x-1} + x + C$

____ 16. Use a table of integrals to evaluate the integral.

$\int x\sqrt{2+2x}\,dx$

a. $\frac{2\sqrt{2}}{15}(3x-2)\sqrt{x+1}+C$

b. $\frac{2}{15}(3x-2)\sqrt{x+1}+C$

c. $\frac{2\sqrt{2}}{15}(2x-1)(x+1)^{3/2}+C$

d. $\frac{2\sqrt{2}}{15}(3x-2)(x+1)^{3/2}+C$

____ 17. Find the integral.

$\int \frac{3x-5}{x^2-2x-3}\,dx$

a. $\ln\left|(x-1)(x+3)^2\right|+C$

b. $\ln\left|(x+1)(x-3)^2\right|+C$

c. $\ln\left|(x-3)(x+1)^2\right|+C$

d. $\ln\left|(x+3)(x-1)^2\right|+C$

____ 18. Use long division to evaluate the integral.

$\int_0^1 \frac{x^3+4x^2-12x+1}{x^2+4x-12}\,dx$

The choices are rounded to 3 decimal places.

a. 0.394
b. –9.606
c. –4.606
d. 5.394
e. 10.394

____ 19. Evaluate the integral.

$$\int 2\left(\frac{x-1}{x^2+2x}\right)dx$$

a. $2\left(\frac{3}{2}\ln(x+2)-\frac{1}{2}\ln x\right)+C$

b. $-2(\ln x - x)+C$

c. $-2\left(\frac{\sqrt{2}}{2}\arctan x\right)+C$

d. $2\left(\frac{1}{3}x-\frac{1}{3}\ln x\right)+C$

e. None of these

____ 20. Evaluate the integral.

$$\int_{\pi/6}^{\pi/3}\frac{2\ln(\tan x)}{5\sin x\cos x}dx$$

a. $\frac{1}{6}(\ln 3)^5$

b. 0

c. $-\frac{1}{5}(\ln 5)^2$

d. $\frac{1}{5}(\ln 3)^2$

e. $-\frac{1}{5}(\ln 3)^2$

Numeric Response

1. Evaluate the integral.

   $\int_0^{1/2} 4x \cos \pi x \, dx$

2. Evaluate the following integral.

   $\int_0^1 \frac{3 \ln 3x}{\sqrt{x}} \, dx$

3. Evaluate the integral.

   $\int \frac{\cos(\ln 7t)}{t} \, dt$

4. Evaluate the integral.

   $\int_0^{\pi/12} 5 \cos^5 6x \, dx$

5. Evaluate the integral.

   $2 \int_0^{\pi/2} \sin^3 \theta \cos^2 \theta \, d\theta$

6. Evaluate the integral.

   $\int_{\pi/3}^{\pi/2} 5 \cot^2 x \, dx$

7. Find the area bounded by the curves $y = 5 \cos x$ and $y = 5 \cos^2 x$ between $x = 0$ and $x = \frac{\pi}{2}$.

8. Find the volume obtained by rotating the region bounded by the given curves about $y = -1$.

   $y = \sin x, \ x = 0, \ x = \pi, \ y = 0$

9. Household electricity is supplied in the form of alternating current that varies from 170 V to –170 V with a frequency of 60 cycles per second (Hz). The voltage is thus given by the function *E(t)*, where *t* is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[E(t)]^2$ over one cycle. Calculate the RMS voltage of household current. Round your answer to the nearest integer.

   $E(t) = 190\sin(120\pi t)$

10. Evaluate the integral using the indicated trigonometric substitution.

    $$\int \frac{dx}{x^2\sqrt{x^2-16}};\ x = 4\sec\theta$$

11. Evaluate the integral to six decimal places.

    $$\int_0^1 \frac{x^3}{\sqrt{25-x^2}}\,dx$$

12. Find the area of the region bounded by the hyperbola $9x^2 - 4y^2 = 36$ and the line $x = 6$.

13. A torus is generated by rotating the circle $x^2 + \left(y-7\right)^2 = 16$ about the *x*-axis. Find the volume enclosed by the torus. Round the answer to the nearest hundredth.

14. A water storage tank has the shape of a cylinder with diameter 10 ft. It is mounted so that the circular cross-sections are vertical. If the depth of the water is 9 ft, what percentage of the total capacity is being used? Round the answer to the nearest tenth.

15. Make a substitution to express the integrand as a rational function and then evaluate the integral.

    $$\int_4^{25} \frac{\sqrt{x}}{x-81}\,dx$$

    Round the answer to four decimal places.

16. Evaluate the integral.

    $$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{2e^{\arctan y}}{1+y^2}\,dy$$

17. Evaluate the integral.

$$\int \frac{x^2}{\left(25-x^2\right)^{3/2}}\,dx$$

18. Evaluate the integral.

$$\int_0^4 \frac{x}{x+25}\,dx$$

19. Evaluate the integral.

$$\int \frac{1}{\sqrt{x+1}+(x+1)\sqrt{x+1}}\,dx$$

20. Evaluate the integral.

$$\int \frac{1+9e^x}{1-e^x}\,dx$$

Short Answer

1. Evaluate the integral.

$$\int_{\pi/2}^{3\pi/4} \frac{x}{\sin^2 x}\,dx$$

2. Evaluate the integral.

$$\int_{\sqrt{e}}^{4e} x^{-2}\ln x\;dx$$

3. Find the integral.

$$\int \cot^6 x \csc^4 x\;dx$$

4. Evaluate the integral using an appropriate trigonometric substitution.

$$\int_1^{\sqrt{2}} \frac{1}{(3+x^2)^{3/2}}\,dx$$

5. Find the integral using an appropriate trigonometric substitution.

$$\int \frac{\sqrt{49x^2-16}}{x}\,dx$$

6. Evaluate the integral.

$$\int_0^1 \frac{3x^3+6x^2+7x+2}{(x+1)^2(x^2+1)}\,dx$$

7. Find the integral.

$$\int \frac{dx}{\sqrt{x^2-4x}}$$

8. Find the integral.

$$\int \frac{3x^2-11x-4}{(x^2+x+1)^2}\,dx$$

9. A corporation is building a complex of homes, offices, stores, schools, and churches in a rural community. As a result of this development, the planners have estimated that the community's population (in thousands) t years from now will be given by

$$P(t)=\frac{3t^2+130t+327}{t^2+6t+45}.$$

What will the average population of the community be over the next 10 years?

10. The region under the graph of

$$y=\frac{8}{x(x+1)}$$

on the interval [1, 2] is revolved about the x-axis. Find the volume of the resulting solid.

Integration Review
Answer Section

MULTIPLE CHOICE

1.	ANS: C	PTS: 1	DIF: Medium	REF: 7.1.24			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.1				
2.	ANS: A	PTS: 1	DIF: Medium	REF: 7.1.38			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.1				
3.	ANS: C	PTS: 1	DIF: Medium	REF: 7.1.12			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.1				
4.	ANS: B	PTS: 1	DIF: Medium	REF: 7.1.41			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.1				
5.	ANS: D	PTS: 1	DIF: Medium	REF: 7.1.2			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.1				
6.	ANS: A	PTS: 1	DIF: Medium	REF: 7.2.1			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.2				
7.	ANS: B	PTS: 1	DIF: Medium	REF: 7.2.2			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.2				
8.	ANS: C	PTS: 1	DIF: Medium	REF: 7.2.33			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.2				
9.	ANS: C	PTS: 1	DIF: Medium	REF: 7.2.47			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.2				
10.	ANS: A	PTS: 1	DIF: Medium	REF: 7.2.65			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.2				
11.	ANS: D	PTS: 1	DIF: Medium	REF: 7.3.1			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.3				
12.	ANS: B	PTS: 1	DIF: Medium	REF: 7.3.2			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.3				
13.	ANS: A	PTS: 1	DIF: Medium	REF: 7.4.37			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				
14.	ANS: B	PTS: 1	DIF: Medium	REF: 7.4.1a			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				
15.	ANS: D	PTS: 1	DIF: Medium	REF: 7.4.20			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				
16.	ANS: D	PTS: 1	DIF: Medium	REF: 7.4.39			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				
17.	ANS: C	PTS: 1	DIF: Medium	REF: 7.4.29			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				
18.	ANS: A	PTS: 1	DIF: Medium	REF: 7.4.16			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				
19.	ANS: A	PTS: 1	DIF: Medium	REF: 7.4.13			
	KEY: 7e	MSC: Bimodal	NOT: Section 7.4				

20. ANS: B PTS: 1 DIF: Medium REF: 7.5.66
KEY: 7e MSC: Bimodal NOT: Section 7.5

NUMERIC RESPONSE

1. ANS: $\dfrac{2\pi - 4}{\pi^2}$

PTS: 1 DIF: Medium REF: 7.1.23 KEY: 7e
MSC: Numerical Response NOT: Section 7.1

2. ANS: $6\ln(3) - 12$

PTS: 1 DIF: Medium REF: 7.1.26 KEY: 7e
MSC: Numerical Response NOT: Section 7.1

3. ANS: $\sin(\ln 7t) + C$

PTS: 1 DIF: Medium REF: 7.1.37 KEY: 7e
MSC: Numerical Response NOT: Section 7.1

4. ANS: $\dfrac{4}{9}$

PTS: 1 DIF: Medium REF: 7.2.9 KEY: 7e
MSC: Numerical Response NOT: Section 7.2

5. ANS: $\dfrac{4}{15}$

PTS: 1 DIF: Medium REF: 7.2.11 KEY: 7e
MSC: Numerical Response NOT: Section 7.2

6. ANS: $\dfrac{5}{\sqrt{3}} - \dfrac{5\pi}{6}$

PTS: 1 DIF: Medium REF: 7.2.35 KEY: 7e
MSC: Numerical Response NOT: Section 7.2

7. ANS: $5 - \dfrac{5\pi}{4}$

PTS: 1 DIF: Medium REF: 7.2.57 KEY: 7e
MSC: Numerical Response NOT: Section 7.2

8. ANS: $\dfrac{\pi^2}{2} + 4\pi$

PTS: 1 DIF: Medium REF: 7.2.61 KEY: 7e
MSC: Numerical Response NOT: Section 7.2

9. ANS: 134

PTS: 1 DIF: Medium REF: 7.2.66 KEY: 7e
MSC: Numerical Response NOT: Section 7.2

10. ANS: $\dfrac{\sqrt{x^2-16}}{16x}+c$

PTS: 1 DIF: Medium REF: 7.3.1 KEY: 7e
MSC: Numerical Response NOT: Section 7.3

11. ANS: 0.050682

PTS: 1 DIF: Medium REF: 7.3.21 KEY: 7e
MSC: Numerical Response NOT: Section 7.3

12. ANS: $6\left(6\sqrt{2}-\ln\left(3+2\sqrt{2}\right)\right)$

PTS: 1 DIF: Medium REF: 7.3.34 KEY: 7e
MSC: Numerical Response NOT: Section 7.3

13. ANS: $2,210.79$

PTS: 1 DIF: Medium REF: 7.3.41 KEY: 7e
MSC: Numerical Response NOT: Section 7.3

14. ANS: 69.8 %

PTS: 1 DIF: Medium REF: 7.3.44 KEY: 7e
MSC: Numerical Response NOT: Section 7.3

15. ANS: -1.207

PTS: 1 DIF: Medium REF: 7.4.44 KEY: 7e
MSC: Numerical Response NOT: Section 7.4

16. ANS: $2e^{\pi/6}-2e^{-\pi/6}$

PTS: 1 DIF: Medium REF: 7.5.7 KEY: 7e
MSC: Numerical Response NOT: Section 7.5

17. ANS: $\dfrac{x}{\sqrt{25-x^2}}-\arcsin\left(\dfrac{x}{5}\right)+C$

PTS: 1 DIF: Medium REF: 7.5.14 KEY: 7e
MSC: Numerical Response NOT: Section 7.5

18. ANS: $4-25\ln 29+50\ln 5$

PTS: 1 DIF: Medium REF: 7.5.24 KEY: 7e
MSC: Numerical Response NOT: Section 7.5

19. ANS: $2\tan^{-1}\sqrt{x+1}+C$

PTS: 1 DIF: Medium REF: 7.5.56 KEY: 7e
MSC: Numerical Response NOT: Section 7.5

20. ANS: $x-10\ln\left(\left|1-e^{x}\right|\right)+C$

PTS: 1 DIF: Medium REF: 7.5.71 KEY: 7e
MSC: Numerical Response NOT: Section 7.5

SHORT ANSWER

1. ANS:
$-\frac{\ln 2}{2}+\frac{3\pi}{4}$

PTS: 1 DIF: Medium REF: 7.1.29 KEY: 7e
MSC: Short Answer NOT: Section 7.1

2. ANS:
$\frac{-\ln 2+3\sqrt{e}-1}{2e}$

PTS: 1 DIF: Medium REF: 7.1.32 KEY: 7e
MSC: Short Answer NOT: Section 7.1

3. ANS:
$-\frac{1}{9}\cot^{9}x-\frac{1}{7}\cot^{7}x+C$

PTS: 1 DIF: Medium REF: 7.2.38 KEY: 7e
MSC: Short Answer NOT: Section 7.2

4. ANS:
$\frac{\sqrt{10}}{15}-\frac{1}{6}$

PTS: 1 DIF: Medium REF: 7.3.7 KEY: 7e
MSC: Short Answer NOT: Section 7.3

5. ANS:
$\sqrt{49x^{2}-16}-4\sec^{-1}\left(\frac{7}{4}x\right)+C$

PTS: 1 DIF: Medium REF: 7.3.19 KEY: 7e
MSC: Short Answer NOT: Section 7.3

6. ANS:
$2\ln 2 + \frac{\pi}{2} - \frac{1}{2}$

PTS: 1 DIF: Medium REF: 7.4.28 KEY: 7e
MSC: Short Answer NOT: Section 7.4

7. ANS:
$\ln\left|x - 2 + \sqrt{x^2 - 4x}\ \right| + C$

PTS: 1 DIF: Medium REF: 7.4.57 KEY: 7e
MSC: Short Answer NOT: Section 7.4

8. ANS:
$2\sqrt{3}\tan^{-1}\left[\frac{\sqrt{3}}{3}(2x+1)\right] + \frac{7}{x^2 + x + 1} + C$

PTS: 1 DIF: Medium REF: 7.4.30 KEY: 7e
MSC: Short Answer NOT: Section 7.4

9. ANS:
9,872

PTS: 1 DIF: Medium REF: 7.4.67 KEY: 7e
MSC: Short Answer NOT: Section 7.4

10. ANS:
$\frac{128\pi}{3}(1 + 3\ln 3 - 6\ln 2)$

PTS: 1 DIF: Medium REF: 7.4.66a KEY: 7e
MSC: Short Answer NOT: Section 7.4